Enrollment No: ______________________ **Exam Seat No:** ______________________

C. U. SHAH UNIVERSITY

Summer Examination-2022

Subject Name : Topology

Subject Code : 5SC01TOP1 **Branch: M.Sc. (Mathematics)**

Semester: 1 **Date: 26/04/2022** **Time: 11:00 To 02:00** **Marks: 70**

Instructions:

(1) Use of Programmable calculator and any other electronic instrument is prohibited.
(2) Instructions written on main answer book are strictly to be obeyed.
(3) Draw neat diagrams and figures (if necessary) at right places.
(4) Assume suitable data if needed.

SECTION – I

Q-1 **Attempt the Following questions.** **[07]**

a. Define: Basis for a topology. **(02)**

b. Let X be topological space and $A, B \subset X$ with $A \subset B$ then prove that $A' \subset B'$. **(02)**

c. Prove that every inclusion map is continuous. **(02)**

d. True/False: Let X be topological space and $A \subset X$ then A is closed if $A' \subset A$. **(01)**

Q-2 **Attempt all questions** **[14]**

a. Let X be a set and $\mathcal{B}$ be a basis for a topology τ on X. Then show that $\tau = \{G : G$ is a union of members of $\mathcal{B}\}$. **(05)**

b. Let X be a set. Define $\tau = \{U \subset X \mid$ either $X - U$ is countable or $X - U = X\}$. Prove that τ is topology on X. **(05)**

c. Let $X = \{a, b, c\}$.Check whether $\tau = \{\emptyset, X, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}\}$ is topology on X or not ? **(04)**

OR

Q-2

a. Let (X_1, τ_1) and (X_2, τ_2) be topological spaces. Let $X = X_1 \times X_2$. Define $\mathcal{B} = \{U_1 \times U_2 \mid U_1 \in \tau_1, U_2 \in \tau_2\}$. Prove that $\mathcal{B}$ is a basis for X. **(05)**

b. Let (X, τ) be topological space and Y be a non-empty subset of X. Then prove that the collection $\tau_y = \{U \cap Y \mid U \in \tau\}$ is topology on Y. **(05)**

c. Let X be a topological space and $A \subset X$ then $x \notin \bar{A}$ if and only if there exists an open set U containing x that does not intersect A. **(04)**

Q-3 **Attempt all questions.** **[14]**

a. Let (X, τ) be topological space and A and B be two subsets of X. Then prove or disprove the following: **(06)**

i) If $A \subset B$ then $\bar{A} \subset \bar{B}$.

ii) $\overline{A \cup B} = \bar{A} \cup \bar{B}$

iii) $\overline{A \cap B} = \bar{A} \cap \bar{B}$.

b. Let R and R_l be the set of real numbers with usual topology and lower limit topology respectively. Let $f: R \to R_l$ defined by $f(x) = x$. Is f continuous function? Justify your answer. **(05)**

c. Compare Lower Limit topology and Usual topology. **(03)**

OR

Q-3

a. Define : Interior point. Let $X = \{a, b, c, d\}$. Define topology on X by $\tau = \{\emptyset, X, \{a\}, \{b\}, \{c\}, \{a, b\}, \{b, c\}, \{a, c\}, \{a, b, c\}\}$. If $A = \{a, d\}$ and $B = \{b, c, d\}$ then find A° and B° . **(05)**

b. State and prove Pasting Lemma. **(05)**

c. Let (X, τ) be topological space. Then show that any finite intersection of open sets in X is open. **(04)**

SECTION – II

Q-4 **Attempt the Following questions.** **[07]**

a Define:Separated Set. **(02)**

b Define: Lindelof space. Give one example of it. **(02)**

c. True/False: Product of second countable space is second countable. **(01)**

d Define: Firstcountability axiom. **(01)**

e. State Urysohn's lemma. **(01)**

Q-5 **Attempt all questions** **[14]**

a. Define: Path Connected. Show that every path connected space is connected. **(05)**

b. State and prove Intermediate value theorem. **(05)**

c. Prove that every subspace of T_2 space is T_2. **(04)**

OR

Q-5

a. Let A be a connected subspace of topological space X. If $A \subset B \subset \bar{A}$ then prove that B is connected. **(05)**

b. Let (X, τ) be disconnected topological space and τ' is finer then τ then prove that (X, τ) is disconnected. **(05)**

c. Let X be a topological space and $A \subset X$. If there is a sequence of points of A converges to x then show that $x \in \bar{A}$. **(04)**

Q-6 **Attempt all questions** **[14]**

a. State and prove Heine – Borel Theorem. **(10)**

b. Prove that continuous image of connected space is connected. **(04)**

OR

Q-6

a. Let X, Y be topological spaces and $f: X \to Y$. Then prove that following are equivalent. **(06)**

(1) f is continuous.

(2) for every subset A of X, then $f(\bar{A}) \subset \overline{f(A)}$

(3) For every closed set B of $Y, f^{-1}(B)$ is closed in X.

b. Prove that every closed subset of compact space is compact. **(04)**

c. Prove that every compact space is locally compact. **(04)**